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Complex degree of coherence measurement for classical statistical fields

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We propose a new method for measuring real and imaginary parts of the complex degree of coherence of a classical field obeying Gaussian statistics. Our method is based on mixing incoherently a coherent Gaussian beam, a local oscillator, and the statistical field. We stress that our approach is especially beneficial for revealing the complex degree of coherence of inhomogeneous two-dimensional fields. As an illustration, we report the complex degree of the coherence measurement of a complex Gaussian-correlated beam. Our method can find applications in image transmission and recovery.

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Spatial coherence is among the fundamental characteristics of light describing the nature of light fluctuations at two or more points within the electric field [1]. The complex degree of coherence (CDOC) of a conventional partially coherent beam (i.e., a Gaussian Schell-model beam) is Gaussian [2,3]. In recent years, it has been firmly established that one can manipulate the CDOC of statistical beams [4–12], and beams with engineered CDOC exhibit many extraordinary properties on propagation, such as self-focusing [5], self-shaping [7,8], self-splitting [9] and self-periodicity [10,11]. The CDOC of partially coherent vortex beams, while Gaussian in the source plane, displays ring dislocations in the far field or in the focal plane of a thin lens. The number of such ring dislocations is determined by the magnitude of its topological charge [13–17]. Further, one can simultaneously determine the magnitude and the sign of the topological charge through measuring the CDOC of a partially coherent vortex beam after having transmitted it through a couple of cylindrical lenses [18]. More recently, we have reported the first experimental generation of optical coherence lattices carrying image information [19]. We have also predicted that the image information can be recovered from the Fourier transform of its CDOC [19]. The presented examples illustrate the importance of statistical light CDOC measurements.

To date, a number of techniques for measuring the CDOC have been proposed. All of them, however, have their limitations. A classic Hanbury Brown and Twiss (HBT) type experiment (intensity–intensity correlation measurements) [8,11] can be used to measure only the magnitude of the CDOC [20]. In principle, wavefront folding interferometry [15,16], Young's double-slit experiment [1,21], Young's boundary diffraction wave technique [22], and the phase-space-based approaches [23,24] allow us to measure both the magnitude and phase of the CDOC. In practice, however, most measurement methods proposed so far face difficulties whenever CDOC measurements of two-dimensional highly inhomogeneous, beam-like fields are required. In particular, wavefront folding interferometry and Young's interference techniques [1,5,16,21] call for a prohibitive number of measurements for the optical beam CDOC recovery. The boundary diffraction wave approach permits the CDOC measurement of a special type of vortex beams with a separable phase [22], while phase space approaches have so far been limited to CDOC measurements of uniform or nearly uniform Gaussian-correlated two-dimensional fields [24]. Thus, an efficient method for CDOC measurements of two-dimensional beams is still missing. The latter will enable one to comprehensively describe realistic partially coherent beam propagation or to recover the image information from the CDOC [19].

In this Letter, we propose a method for determining the real and imaginary parts of the CDOC of classical random beams obeying Gaussian statistics through an incoherent superposition of such beams with a strong coherent beam. In our method, the CDOC magnitude is determined directly through measuring its intensity–intensity correlations, while the real part of the CDOC is determined through measuring the intensity–intensity correlation function of a combined beam. The novelty of our method is in tailoring the strong coherent beam profile to ensure accuracy of the CDOC measurement.

To illustrate our method, we recover the CDOC of a complex Gaussian-correlated beam.

Figure 1 shows the schematics for the incoherent superposition of a partially coherent beam (PCB) and a coherent beam (CB). $\mathbf{r} \equiv (x, y)$ is an arbitrary coordinate in the source plane. The proposed setup is essentially the one used for homodyne detection in quantum optics [1]. A strong coherent field (e.g., Gaussian beam) is known as a local oscillator (LO), and the input partially coherent beam is referred to as the signal. The electric field of the signal and the LO in the source plane are $\epsilon_s(\mathbf{r})$ and $|\epsilon_{\mathrm{LO}}(\mathbf{r})| \exp(-i\phi)$, respecitvely, with ϕ being the phase of the LO. The electric field of the BS output is then

$$E(\mathbf{r}) = \epsilon_s(\mathbf{r}) + |\epsilon_{\mathrm{LO}}(\mathbf{r})| \exp(-i\phi). \quad (1)$$

It follows at once from Eq. (1) that the output intensity distribution is given by the expression

$$\begin{aligned} I(\mathbf{r}) &\equiv |E(\mathbf{r})|^2 \\ &= I_s(\mathbf{r}) + I_{\mathrm{LO}}(\mathbf{r}) + \epsilon_s^*(\mathbf{r})|\epsilon_{\mathrm{LO}}(\mathbf{r})| \exp(-i\phi) + \mathrm{c.c.}, \end{aligned} \quad (2)$$

where c.c. stands for the complex conjugate. The intensities of the signal and the LO are expressed as $I_s(\mathbf{r}) = |\epsilon_s(\mathbf{r})|^2$ and $I_{\mathrm{LO}}(\mathbf{r}) = |\epsilon_{\mathrm{LO}}(\mathbf{r})|^2$, respectively. The intensity–intensity correlation function of the output is defined as [8,12]

$$G_\phi^{(2)}(\mathbf{r}_1, \mathbf{r}_2) = \langle I(\mathbf{r}_1) I(\mathbf{r}_2) \rangle, \quad (3)$$

where $\mathbf{r}_1 \equiv (x_1, y_1)$ and $\mathbf{r}_2 \equiv (x_2, y_2)$ are two arbitrary radius vectors in the source plane; $\langle \cdot \rangle$ denotes the ensemble average. Assuming the local oscillator intensity dwarfs the signal intensity, $I_s \ll I_{\mathrm{LO}}$, and keeping the terms only linear or higher order in I_{LO}, we obtain for the intensity–intensity correlation the expression

$$\begin{aligned} G_\phi^{(2)}(\mathbf{r}_1, \mathbf{r}_2) &= \langle I(\mathbf{r}_1) I(\mathbf{r}_2) \rangle \\ &\simeq I_{\mathrm{LO}}(\mathbf{r}_1) I_{\mathrm{LO}}(\mathbf{r}_2) + I_{\mathrm{LO}}(\mathbf{r}_1)\langle I_s(\mathbf{r}_2)\rangle + I_{\mathrm{LO}}(\mathbf{r}_2)\langle I_s(\mathbf{r}_1)\rangle \\ &+ 2\sqrt{I_{\mathrm{LO}}(\mathbf{r}_1) I_{\mathrm{LO}}(\mathbf{r}_2)\langle I_s(\mathbf{r}_1)\rangle\langle I_s(\mathbf{r}_2)\rangle}\, \mathrm{Re}[\gamma(\mathbf{r}_1, \mathbf{r}_2)], \end{aligned} \quad (4)$$

where $\gamma(\mathbf{r}_1, \mathbf{r}_2)$ is the CDOC of the signal and "Re" denotes the real part. In deriving Eq. (4), we assumed that the signal field has a fast fluctuating phase such that the first- and higher order phase-sensitive correlation functions vanish, $\langle \epsilon_s(\mathbf{r}) \rangle = 0$, $\langle \epsilon_s^*(\mathbf{r}_1) \epsilon_s^*(\mathbf{r}_2) \rangle = \langle \epsilon_s(\mathbf{r}_1) \epsilon_s(\mathbf{r}_2) \rangle = 0$. Importantly, this result is valid for any classical statistical light fields, including multimode laser light when only a few modes are exited, LEDs, and random lasers among others that generate optical fields with non-Gaussian statistics.

Hereafter, it proves convenient to introduce a normalized intensity–intensity correlation function $g_\phi^{(2)}(\mathbf{r}_1, \mathbf{r}_2) = G_\phi^{(2)}(\mathbf{r}_1, \mathbf{r}_2)/[I_{\mathrm{LO}}(\mathbf{r}_1) I_{\mathrm{LO}}(\mathbf{r}_2)]$, which reads

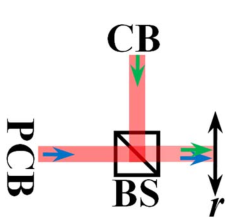

Fig. 1. Schematic for the incoherent superposition of a PCB and a CB. BS, beam splitter.

$$g_\phi^{(2)}(\mathbf{r}_1, \mathbf{r}_2) \simeq 1 + I(\mathbf{r}_1) + I(\mathbf{r}_2) + 2\sqrt{I(\mathbf{r}_1) I(\mathbf{r}_2)}\, \mathrm{Re}[\gamma(\mathbf{r}_1, \mathbf{r}_2)]. \quad (5)$$

Here, the normalized signal to LO intensity is introduced as $I(\mathbf{r}) = I_s(\mathbf{r})/I_{\mathrm{LO}}(\mathbf{r})$. It is convenient to choose the LO such that its intensity profile coincides with the signal. It then follows that $I(\mathbf{r}) = I_0 = \mathrm{const}$, where I_0 is a ratio of peak intensities of the signal and LO. As $I_0 \ll 1$, we can infer from Eq. (5) the simple expression for the real part of the degree of coherence as

$$\mathrm{Re}[\gamma(\mathbf{r}_1, \mathbf{r}_2)] \simeq [g_\phi^{(2)}(\mathbf{r}_1, \mathbf{r}_2) - 1]/2I_0. \quad (6)$$

Thus, we can determine the real part of the CDOC of the signal by measuring the intensity–intensity correlation function of the combined beam. In traditional methods (e.g., HBT-type experiment), the modulus of the signal CDOC can be measured by assuming the light field obeys Gaussian statistics [1,8,12] such that

$$g_s^{(2)}(\mathbf{r}_1, \mathbf{r}_2) = \frac{\langle I_s(\mathbf{r}_1) I_s(\mathbf{r}_2) \rangle}{\langle I_s(\mathbf{r}_1) \rangle \langle I_s(\mathbf{r}_2) \rangle} = 1 + |\gamma(\mathbf{r}_1, \mathbf{r}_2)|^2. \quad (7)$$

It follows at once from Eqs. (6) and (7), which are the cornerstone of our procedure, that both real and imaginary parts of the CDOC can be recovered. We stress, though, that while Eq. (6) holds for arbitrary statistical fields, Eq. (7) is limited to the fields obeying Gaussian statistics, making our methodology ultimately applicable to Gaussian statistical fields.

Let us illustrate our method by determining the CDOC of a complex Gaussian-correlated (CGC) beam. The CGC beam (i.e., signal) is realized by performing an optical "Fourier transform" of the Gaussian beam array with the intensity distribution as

$$I(\mathbf{s}) = \frac{1}{N} \sum_{n=1}^{N} \exp\left[\frac{(s_x + a_n)^2 + (s_y + b_n)^2}{\omega_0^2/2}\right], \quad (8)$$

with a thin lens. Here, $\mathbf{s} = (s_x, s_y)$ is an arbitrary coordinate on the Fourier transform plane, N is a positive integer, and ω_0 is a beam width. a_n and b_n are the displacements from the origin coordinate in x and y directions, respectively. The CDOC of the CGC beam in the source plane can be expressed as [1,8]

$$\begin{aligned} \gamma(\mathbf{r}_1, \mathbf{r}_2) &= \frac{1}{\lambda^2 f^2} \int I(\mathbf{s}) \exp\left[-\frac{i2\pi \mathbf{s} \cdot (\mathbf{r}_1 - \mathbf{r}_2)}{\lambda f}\right] \mathrm{d}^2\mathbf{s} \\ &= \sum_{n=1}^{N} \exp\left\{-\frac{(\mathbf{r}_1 - \mathbf{r}_2)^2}{2\delta_0^2} + \frac{2i\pi}{\lambda f}[a_n(x_1 - x_2) + b_n(y_1 - y_2)]\right\}, \end{aligned} \quad (9)$$

where $\delta_0 = \lambda f/(\pi\omega_0)$ is a transverse coherent length of the CGC beam, with f being the focal length of a lens performing the Fourier transform. The intensity distribution of the CGC beam in the source plane is

$$I_s(\mathbf{r}) = C_s \exp\left(-\frac{\mathbf{r}^2}{2\sigma_s^2}\right), \quad (10)$$

where σ_s and C_s are the beam waist size and the amplitude parameter of the CGC beam, respectively. The Gaussian local oscillator has the intensity distribution in the source plane as

$$I_{\mathrm{LO}}(\mathbf{r}) = C_{\mathrm{LO}} \exp\left(-\frac{\mathbf{r}^2}{2\sigma_{\mathrm{LO}}^2}\right), \quad \textbf{(11)}$$

where C_{LO} and σ_{LO} are the amplitude parameter and the beam waist size of LO, respectively. The amplitude parameters have the relationship $C_s + C_{\mathrm{LO}} = 1$. The LO phase is taken here as $\phi = \pi/2$ caused by the BS reflection [20].

In the following experiment, we set $N = 3$, and (a_1, b_1), (a_2, b_2), and (a_3, b_3) to be $(0, 0)$, $(0, 2\omega_0)$, and $(2\omega_0, 0)$, respectively. The experimental setup is shown in Fig. 2.

The experimental procedure can be divided into two parts. Part I involved mixing the CGC beam and LO. A linearly polarized diode-pumped solid-state laser (DPL) light beam of wavelength $\lambda = 532$ nm is expanded by a beam expander (BE) and split into two components by a beam splitter (BS_1). The reflected beam was modulated by a spatial light modulator (SLM), controlled by a personal computer (PC_1), and contained a phase mask. The selected Gaussian beam array (GBA) by a circular aperture (CA) was reflected by a mirror M_1 and focused onto a rotating ground glass disk (RGGD) by a thin lens L_1. The scattered light from the RGGD was then collimated by a thin lens L_2 and shaped by a Gaussian amplitude filter (GAF), producing a CGC beam. The generated CGC beam was mixed with the local oscillator that was transmitted through BS_1 and reflected by the mirror M_2 at BS_2.

Part II involved the intensity and CDOC distribution measurement of the CGSCM beam in the source plane. The combined beam at BS_2 passing through a $4f$ image system consisting of a thin lens L_3 ($f_3 = 10$ cm) was then received by a charge-coupled device (CCD) with pixel size 4.4 μm × 4.4 μm. The intensity distributions of the CGC, LO, and the combined beam were captured by the CCD (each 6000 frames) and then sent to a personal computer (PC_2) and processed by the commercial software MATLAB.

Figure 3 shows the experimental results of the CGC beam intensity distribution in the source plane. It was obtained from the normalized intensity distribution of the captured 6000 frames by the CCD. The corresponding Gaussian fit in Fig. 3(b) shows the beam waist size of the CGC beam to be $\sigma_s = 0.5$ mm; the beam size is fixed hereafter. The beam waist size of the LO is adjusted by the BE and set to $\sigma_{\mathrm{LO}} = \sigma_s$, which we do not show here.

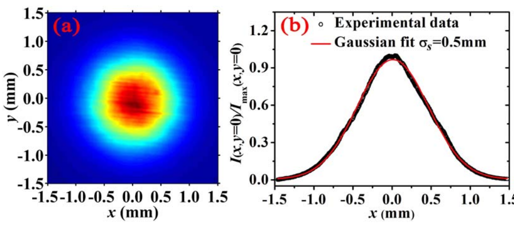

Fig. 3. Experimental results for the CGC beam intensity distribution in the source plane. (a) The contour plot, and (b) the corresponding cross line with a Gaussian fit $\sigma_s = 0.5$ mm.

Figure 4 shows theoretical and experimental results for the normalized intensity–intensity correlation function distribution of the CGC beam [Figs. 4(b-1)–4(c-1)] and the combined beam [Figs. 4(a-2)–4(c-2)] with $C_{\mathrm{LO}} = 0.91$ in the source plane. The coherence length is $\delta_0 = 0.08$ mm, and the reference point of the intensity–intensity correlation function is fixed at the beam center (i.e., $\mathbf{r}_2 = 0$) hereafter. We can infer from Fig. 4 that the normalized intensity–intensity correlation function of the combined beam in the source plane is modified upon mixing with the LO [Figs. 4(a-2)–4(c-2)] as compared to that without the LO [Figs. 4(a–1)–4(c–1)]. This phenomenon could be explained by analyzing Eq. (6). As $C_{\mathrm{LO}} \gg C_s$ (i.e., $I_0 \ll 1$) in the experiment, the normalized intensity–intensity correlation function of the combined beam in the source plane depends on the real part of the signal CDOC.

Figure 5 shows the theoretical and experimental results for the real part [Figs. 5(a-1)–5(c-1)] and the square modulus of the imaginary part [Figs. 5(a-2)–5(c-2)] of the CDOC of the generated CGCSM beam in the source plane, where the corresponding parameters are the same as those in Fig. 4. We can infer from Fig. 5 that the real part and the square modulus of the imaginary part of the CDOC of the generated CGCSM beam differ substantially. That is, the measurements of the real and the imaginary parts have great importance and both of them could be used for information transfer. Our experimental results agree reasonably well with the theoretical simulations.

In summary, we proposed a method for separately measuring real and imaginary parts of the complex degree of coherence of classical light obeying Gaussian statistics. Our method

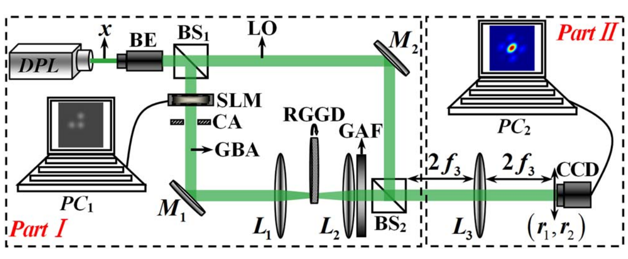

Fig. 2. Experimental setup for the superposition of the CGCSM beam and GB and the measurement of the CDOC. DPL, diode-pumped solid-state laser; BE, beam expander; BS_1 and BS_2, beam splitters; SLM, spatial light modulator; CA, circular aperture; M_1 and M_2, reflect mirrors; L_1, L_2, and L_3, thin lenses; RGGD, rotating ground glass dish; GAF, Gaussian amplitude filter; CCD, charge-coupled device; PC_1 and PC_2, personal computers; GB, Gaussian beam; GBA, Gaussian beam array.

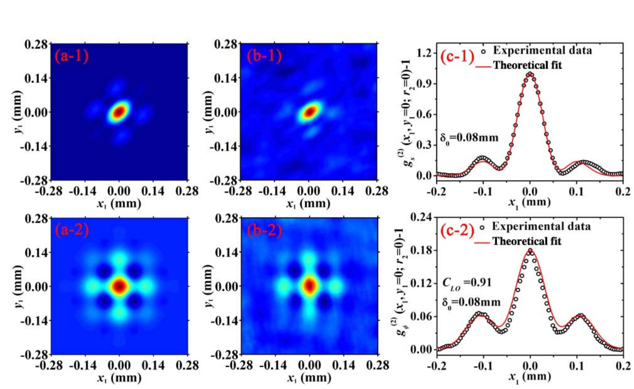

Fig. 4. Theoretical and experimental results for the normalized intensity–intensity correlation function of (a-1)–(c-1) the CGC beam and (a-2)–(c-2) the combined beam with $C_{\mathrm{LO}} = 0.91$ in the source plane.

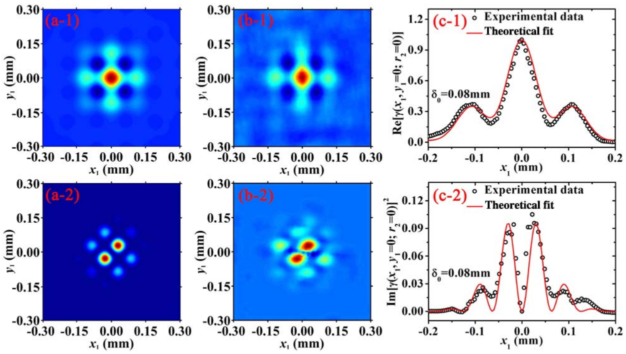

Fig. 5. Theoretical and experimental results for the real part (a-1)–(c-1) and (a-2)–(c-2) the square modulus of the imaginary part of the CDOC of the CGC beam.

involves mixing a statistical field with a strong coherent local oscillator. We illustrated the power of our method by measuring the CDOC of a complex Gaussian-correlated beam. Our method provides a convenient way to measure the CDOC of partially coherent fields and it can find applications in image transmission and recovery.

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